Introduction to the MSSM

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Abstract

These lecture notes are based on a first course on the Minimal Supersymmetric Standard Model. The level of the notes is introductory and pedagogical. Standard Model, basic supersymmetry algebra and its representations are considered as prerequisites. The topics covered include particle content, structure of the lagrangian, supersymmetry breaking soft terms, electroweak symmetry breaking and the sparticle mass spectrum. Popular supersymmetry breaking models like minimal supergravity and gauge mediation are also introduced.

arXiv:1201.0334v1 [hep-ph] 1 Jan 2012

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I. PREREQUISITES

These lectures[1] are devised as an introduction to the Minimal Supersymmetric Standard Model. In the course of these lectures, we will introduce the basic features of the Supersymmetric Standard Model, the particle content, the structure of the lagrangian, feynman rules, supersymmetric breaking soft terms, Electroweak symmetry breaking and the mass spectrum of the MSSM. Supersymmetry is a vast subject and these lectures are definitely not a comprehensive review and in fact, they are also not what one could term as an introduction to supersymmetric algebra and supersymmetric gauge theories. The prerequisites for this course are a good knowledge of the Standard Model and also supersymmetry say, at the level of first eight chapters of Wess and Bagger[1]: supersymmetry transformations, representations, superfields, supersymmetric gauge theories and elements of supersymmetry breaking. It is strongly recommended that readers keep a text book on basic course of supersymmetry [2–4] with them all the time for consulting, while going through this lecture notes.

The lectures are organised as follows : in the next section, we will give a lightening introduction to the Standard Model and the structure of its lagrangian. The reason for this being that we will like to introduce the MSSM (Minimal Supersymmetric Standard Model) in a similar organisational fashion, which makes it easier to remember the MSSM lagrangian - as well as arranging the differences and similarities , one expects in the supersymmetric theories in a simpler way, if possible. The next section would introduce the basic form of the MSSM lagrangian - the three functions of the chiral/vector superfields - the superpotential, the Kahler potential and the field strength superfield and the particle spectrum. The fourth section will be devoted to R-parity and some sample feynman rules. Supersymmetry breaking and electroweak symmetry breaking will be introduced in section 5 and the physical supersymmetric particle mass spectrum will be done in section 6. Higgs sector will be reviewed in section 7, while we close with some 'standard' models of supersymmetry breaking in section 8.

Finally for the students not completely familiar with Standard Model, we point out at some references with increasing order of difficulty in reading and requirements of pre-requisites. These are : (a) Aitchison and Hey[5], Gauge Field Theories, Vol I and Vol II (b)A good functional introduction to field theory required for understanding Standard Model can be found in : M. Srednicki, Quantum Field Theory [6] (c) M. E. Peskin and D. Schroeder, Quantum Field Theory

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[1] Based on lectures presented at SERC school, held at IIT-Bombay, Mumbai.

[7] (d) E. Abers and B. W. Lee, Physics Reports on Gauge Field Theories [8] (e) T. Cheng and L. Li, Gauge theory of elementary particle physics[9] (f) S. Weinberg, Quantum Theory of Fields, Vol I -II[10], (g) S. Pokorski, Gauge Field Theories [11] and (h) Donoghue, Golowich and Holstein, Dynamics of the Standard Model [12].

II.   STEP 0 : A LIGHTENING RECAP OF THE STANDARD MODEL

The Standard Model (SM) is a spontaneously broken Yang-Mills quantum field theory describing the strong and electroweak interactions. The theoretical assumption on which the Standard Model rests on is the principle of local gauge invariance with the gauge group given by

$$G_{SM} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y, \tag{1}$$

where the subscript c stands for color, L stands for the 'left-handed' chiral group whereas Y is the hypercharge. The particle spectrum and their transformation properties under these gauge groups are given as,

$$Q_i \equiv \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix} \sim \left(3, \, 2, \, \frac{1}{6}\right) \qquad U_i \equiv u_{Ri} \sim \left(\bar{3}, \, 1, \, \frac{2}{3}\right)$$

$$D_i \equiv d_{Ri} \sim \left(\bar{3}, \, 1, \, -\frac{1}{3}\right)$$

$$L_i \equiv \begin{pmatrix} \nu_{Li} \\ e_{Li} \end{pmatrix} \sim \left(1, \, 2, \, -\frac{1}{2}\right) \qquad E_i \equiv e_{Ri} \sim (1, \, 1, \, -1)$$

In the above i stands for the generation index, which runs over the there generations $i = 1, 2, 3$. Q_i represents the left handed quark doublets containing both the up and down quarks of each generation. Similarly, L_i represents left handed lepton doublet, U_i, D_i, E_i represent right handed up-quark, down-quark and charged lepton singlets respectively. The numbers in the parenthesis represent the transformation properties of the particles under G_{SM} in the order given in eq.(1). For example, the quark doublet Q transforms a triplet (3) under $SU(3)$ of strong interactions, a doublet (2) under weak interactions gauge group and carry a hypercharge ($Y/2$) of 1/6 [2]. In addition to the fermion spectra represented above, there is also a fundamental scalar called Higgs

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[2] Note that the hypercharges are fixed by the Gellman-Nishijima relation $Y/2 = Q - T_3$, where Q stands for the charge of the particle and T_3 is the eigenvalue of the third generation of the particle under $SU(2)$.

whose transformation properties are given as

$$H \equiv \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \sim (1, \ 2, \ 1/2) \,. \tag{2}$$

However, the requirement of local gauge invariance will not be fulfilled unless one includes the gauge boson fields also. Including them, the total lagrangian with the above particle spectrum and gauge group can be represented as,

$$\mathcal{L}_{SM} = \mathcal{L}_F + \mathcal{L}_{YM} + \mathcal{L}_{yuk} + \mathcal{L}_S. \tag{3}$$

The fermion part $\mathcal{L}_F$ gives the kinetic terms for the fermions as well as their interactions with the gauge bosons. It is given as,

$$\mathcal{L}_F = i\bar{\Psi}\gamma^\mu \mathcal{D}_\mu \Psi, \tag{4}$$

where Ψ represents all the fermions in the model,

$$\Psi = (Q_i \ U_i, \ D_i, \ L_i, \ E_i) \tag{5}$$

where $\mathcal{D}_\mu$ represents the covariant derivative of the field given as,

$$\mathcal{D}_\mu = \partial\mu - ig_s G_\mu^A \lambda^A - i\frac{g}{2} W_\mu^I \tau^I - ig' B_\mu Y \tag{6}$$

Here $A = 1, .., 8$ with G_μ^A representing the $SU(3)_c$ gauge bosons, $I = 1, 2, 3$ with W_μ^I representing the $SU(2)_L$ gauge bosons. The $U(1)_Y$ gauge field is represented by B_μ. The kinetic terms for the gauge fields and their self interactions are given by,

$$\mathcal{L}_{YM} = -\frac{1}{4} G^{\mu\nu A} G_{\mu\nu}^A - \frac{1}{4} W^{\mu\nu I} W_{\mu\nu}^I - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \tag{7}$$

with

$$\begin{aligned} G_{\mu\nu}^A &= \partial_\mu G_\nu^A - \partial_\nu G_\mu^A + g_s \ f_{ABC} G_\mu^B G_\nu^C \\ F_{\mu\nu}^I &= \partial_\mu W_\nu^I - \partial_\nu W_\mu^I + g \ f_{IJK} W_\mu^J W_\nu^K \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \end{aligned} \tag{8}$$

where $f_{ABC(IJK)}$ represent the structure constants of the $SU(3)(SU(2))$ group.

In addition to the gauge bosons, the fermions also interact with the Higgs boson, through the dimensionless Yukawa couplings given by

$$\mathcal{L}_{yuk} = h_{ij}^u \bar{Q}_i U_j \tilde{H} + h_{ij}^d \bar{Q}_i D_j H + h_{ij}^e \bar{L}_i E_j H + H.c \tag{9}$$

where $\tilde{H} = i\sigma^2 H^\star$. These couplings are responsible for the fermions to attain masses once the gauge symmetry is broken from $G_{SM} \rightarrow SU(3)_c \times U(1)_{em}$. This itselves is achieved by the scalar part of the lagrangian which undergoes spontaneous symmetry breakdown. The scalar part of the lagrangian is given by,

$$\mathcal{L}_S = (\mathcal{D}_\mu H)^\dagger \mathcal{D}_\mu H - V(H), \qquad (10)$$

where

$$V(H) = \mu^2 H^\dagger H + \lambda \left(H^\dagger H \right)^2 \qquad (11)$$

For $\mu^2 < 0$, the Higgs field attains a vacuum expectation value (*vev*) at the minimum of the potential. The resulting goldstone bosons are 'eaten away' by the gauge bosons making them massive through the so-called Higgs mechanism. Only one degree of the Higgs field remains physical, the only scalar particle of the SM - the Higgs boson. The fermions also attain their masses through their Yukawa couplings, once the Higgs field attains a *vev*. The only exception is the neutrinos which do not attain any mass due to the absence of right handed neutrinos in the particle spectrum and thus the corresponding Yukawa couplings. Finally, the Standard Model is renormalisable and anomaly free. We would also insist that the Supersymmetric version of the Standard Model keeps these features of the Standard Model intact.

1. *Think it Over*

Here are some important aspects of the Standard Model which have not found a mention in the above. These are formulated in some sort of a problem mode, which would require further study.

- What is the experiment that showed that there are only three generations of particles in the Standard Model ? Can one envisage a fourth generation ? If so, what are the constraints this generation of particles expected to satisfy ?

- The gauge bosons 'mix' at the tree level by an angle $\tan \theta_W = (\frac{g'}{g})^2$. What happens at the 1-loop level ? However are the relevant observables classified ? Some relevant information can be found at[13].

- What are the theoretical limits on the Higgs boson mass ? How sensitive is the upper bound on the Higgs mass from precision measurements to the top quark mass ? What is the lower limit on the Higgs mass from the LEP experiment ? Some relevant information can be found at[14].

- The LHC experiment has been rapidly constraining the allowed parameter space of the Higgs boson. For latest information have a look at [15].

- What is the CKM mixing ? How well are these angles measured ? What is the present status after the results from various B-factories about the CP phase in the SM ? What is the analogous mixing in the leptonic sector called ? In comparison to the CKM matrix, how well are these angles measured ?

III.   STEP 1 : PARTICLE SPECTRUM OF MSSM

What we aim to build over the course of next few lectures is a supersymmetric version of the Standard Model, which means the lagrangian we construct should not only be gauge invariant under the Standard Model gauge group G_{SM} but also now be supersymmetric invariant. Such a model is called Minimal Supersymmetric Standard Model with the the word 'Minimal' referring to minimal choice of the particle spectrum required to make it work. Furthermore, we would also like the MSSM to be renormalisable and anomaly free, just like the Standard Model is.

Before we proceed to discuss about the particle spectrum, let us remind ourselves that ordinary quantum fields are upgraded in supersymmetric[3] theories to so-called supermultiplets or superfields[4]. Given that supersymmetry transforms a fermion into a boson and vice-versa, supermultiplets or superfields are multiplets which collect fermion-boson pairs which transform in to each other. We will deal with two kinds of superfields - vector superfields and chiral superfields. A chiral superfield[5] contains a weyl fermion, a scalar and and an auxiliary scalar field generally denoted by F. A vector superfield contains a spin 1 boson, a spin 1/2 fermion and an auxiliary scalar field called D.

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[3] All through this set of lectures, whenever we mention supersymmetry we mean N=1 SUSY ; only one set of SUSY generators.

[4] Superfields are functions (fields) written over a 'superspace' made of ordinary space (x_μ) and two fermionic 'directions' $(\theta, \bar{\theta})$; they are made up of quantum fields whose spins differ by 1/2. To build interaction lagrangians one normally resorts to this formalism, originally given by Salam and Strathdee[16], as superfields simplify addition and multiplication of the representations. It should be noted however that the component fields may always be recovered from superfields by a power series expansion in grassman variable, θ.

A chiral superfield has an expansion :

$$\Phi = \phi + \sqrt{2}\theta\psi + \theta\theta F, \tag{12}$$

where ϕ is the scalar component, ψ, the two component spin 1/2 fermion and F the auxiliary field. A vector superfield in (Wess-Zumino gauge) has an expansion :

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D \tag{13}$$

[5] Here we are presenting the particle content in the off-shell formalism.

The minimal supersymmetric extension of the Standard Model is built by replacing every standard model matter field by a chiral superfield and every vector field by a vector superfield. Thus the existing particle spectrum of the Standard Model is doubled. The particle spectrum of the MSSM and their transformation properties under G_{SM} is given by,

$$Q_i \equiv \begin{pmatrix} u_{L_i} & \tilde{u}_{L_i} \\ d_{L_i} & \tilde{d}_{L_i} \end{pmatrix} \sim \left(3, \ 2, \ \frac{1}{6} \right) \qquad U_i^c \equiv \begin{pmatrix} u_i^c & \tilde{u}_i^c \end{pmatrix} \sim \left(\bar{3}, \ 1, \ -\frac{2}{3} \right)$$

$$D_i \equiv \begin{pmatrix} d_i^c & \tilde{d}_i^c \end{pmatrix} \sim \left(\bar{3}, \ 1, \ \frac{1}{3} \right)$$

$$L_i \equiv \begin{pmatrix} \nu_{L_i} & \tilde{\nu}_{L_i} \\ e_{L_i} & \tilde{e}_{L_i} \end{pmatrix} \sim \left(1, \ 2, \ -\frac{1}{2} \right) \qquad E_i \equiv \begin{pmatrix} e_i^c & \tilde{e}_i^c \end{pmatrix} \sim (1, \ 1, \ 1)$$

$$(14)$$

The scalar partners of the quarks and the leptons are typically named as 's'quarks and 's'leptons. Together they are called sfermions. For example, the scalar partner of the top quark is known as the 'stop'. In the above, these are represented by a 'tilde' on their SM counterparts. As in the earlier case, the index i stands for the generation index.

There are two distinct features in the spectrum of MSSM : (a) Note that we have used the conjugates of the right handed particles, instead of the right handed particles themselves. There is no additional conjugation on the superfield itselves, the c in the superscript just to remind ourselves that this chiral superfield is made up of conjugates of SM quantum fields. In eq.(14), $u^c = u_R^\dagger$ and $\tilde{u}^c = \tilde{u}_R^\star$. This way of writing down the particle spectrum is highly useful for reasons to be mentioned later in this section. Secondly (b) At least two Higgs superfields are required to complete the spectrum - one giving masses to the up-type quarks and the other giving masses to the down type quarks and charged leptons. As mentioned earlier, this is the minimal number of Higgs particles required for the model to be consistent from a quantum field theory point of view[6]. These two Higgs superfields have the following transformation properties under G_{SM}:

$$H_1 \equiv \begin{pmatrix} H_1^0 & \tilde{H}_1^0 \\ H_1^- & \tilde{H}_1^- \end{pmatrix} \sim \left(1, \ 2, \ -\frac{1}{2} \right)$$

$$H_2 \equiv \begin{pmatrix} H_2^+ & \tilde{H}_2^+ \\ H_2^0 & \tilde{H}_2^0 \end{pmatrix} \sim \left(1, \ 2, \ \frac{1}{2} \right) \qquad (15)$$

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[6] The Higgs field has a fermionic partner, higgsino which contributes to the anomalies of the SM. At least two such fields with opposite hyper-charges $(U(1)_Y)$ should exist to cancel the anomalies of the Standard Model.

The Higgsinos are represented by a ˜ on them. This completes the matter spectrum of the MSSM. Then there are the gauge bosons and their super particles. Remember that in supersymmetric theories, the gauge symmetry is imposed by the transformations on matter superfields as :

$$\Phi' = e^{i\Lambda_l t_l}\Phi \tag{16}$$

where Λ_l is an arbitrary chiral superfield and t_l represent the generators of the gauge group which are l in number and the index l is summed over[7]. The gauge invariance is restored in the kinetic part by introducing a (real) vector superfield, V such that the combination

$$\Phi^\dagger e^{gV}\Phi \tag{17}$$

remains gauge invariant. For this to happen, the vector superfield V itselves transforms under the gauge symmetry as

$$\delta V = i(\Lambda - \Lambda^\dagger) \tag{18}$$

The supersymmetric invariant kinetic part of the lagrangian is given by:

$$\mathcal{L}_{kin} = \int d\theta^2 d\bar{\theta}^2 \Phi^\dagger e^{gV}\Phi = \Phi^\dagger e^{gV}\Phi|_{\theta\theta\bar{\theta}\bar{\theta}} \tag{19}$$

In the MSSM, corresponding to three gauge groups of the SM and for each of their corresponding gauge bosons, we need to add a vector superfield which transforms as the adjoint under the gauge group action. Each vector superfield contains the gauge boson and its corresponding super partner called gaugino. Thus in MSSM we have the following vector superfields and their corresponding transformation properties under the gauge group, completing the particle spectrum of the MSSM:

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\begin{aligned}
V_s^A &: \left(G^{\mu A} \ \tilde{G}^A \right) &\sim \quad (8,1,0) \\
V_W^I &: \left(W^{\mu I} \ \tilde{W}^I \right) &\sim \quad (1,3,0) \\
V_Y &: \left(B^\mu \ \tilde{B} \right) &\sim \quad (1,1,0)
\end{aligned} \tag{20}
$$

The G's (G and $\tilde{G}$) represent the gluonic fields and their superpartners called gluinos, the index A runs from 1 to 8. The W's are the $SU(2)$ gauge bosons and their superpartners 'Winos', the index I taking values from 1 to 3 and finally Bs represents the $U(1)$ gauge boson and its superpartner 'Bino'. Together all the superpartners of the gauge bosons are called 'gauginos'. This completes the particle spectrum of the MSSM.

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[7] To be more specific, t_l is just a number for the abelian groups. For non-abelian groups, t_l is a matrix and so is Λ_l, with $\Lambda_{ij} = t_{ij}^l \Lambda_l$ Note that V is also becomes a matrix in this case.

IV.   STEP 2: THE SUPERPOTENTIAL AND R-PARITY

The supersymmetric invariant lagrangian is constructed from functions of superfields. In general there are three functions which are: (a) The Kähler potential, K, which is a real function of the superfields (b) The superpotential W, which is a holomorphic (analytic) function of the superfields, and (c) the gauge kinetic function $f_{\alpha\beta}$ which appears in supersymmetric gauge theories. This is the coefficient of the product of field strength superfields, $\mathcal{W}_\alpha \mathcal{W}^\beta$. The field strength superfield is derived from the vector superfields contained in the model. $f_{\alpha\beta}$ determines the normalisation for the gauge kinetic terms. In MSSM, $f_{\alpha\beta} = \delta_{\alpha\beta}$. The lagrangian of the MSSM is thus given in terms of G_{SM} gauge invariant functions K, W and add the field strength superfield $\mathcal{W}$, for each of the vector superfields in the spectrum.

The gauge invariant Kähler potential has already been discussed in the eqs.(19). For the MSSM case, the Kähler potential will contain all the three vector superfields corresponding to the G_{SM} given in the eq.(20). Thus we have :

$$\mathcal{L}_{kin} = \int d\theta^2 d\bar\theta^2 \sum_{SU(3),SU(2),U(1)} \Phi_\beta^\dagger \, e^{gV} \Phi_\beta \tag{21}$$

where the index β runs over all the matter fields $\Phi_\beta = \{Q_i, U_i^c, D_i^c, L_i, e_i^c, H_1, H_2\}$[8] in appropriate representations. Corresponding to each of the gauge groups in G_{SM}, all the matter fields which transform non-trivially under this gauge group[9] are individually taken and the grassman $(d\theta^2 d\bar\theta^2)$ integral is evaluated with the corresponding vector superfields in the exponential[10]. After expanding and evaluating the integral, we get the lagrangian which is supersymmetric invariant in terms of the ordinary quantum fields - the SM particles and the superparticles. This part of the lagrangian would give us the kinetic terms for the SM fermions, kinetic terms for the sfermions and their interactions with the gauge bosons and in addition also the interactions of the type: fermion-sfermion-gaugino which are structurally like the Yukawa interactions ($ff\phi$), but carry gauge couplings. Similarly, for the Higgs fields, this part of the lagrangian gives the kinetic terms for the Higgs fields and their fermionic superpartners Higgsinos and the interaction of the gauge

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[8] The indices i, j, k always stand for the three generations through out this notes, taking values between 1 and 3.

[9] As given in the list of representations in eqs. (14,15)

[10] Remember that the function e^{gV} truncates at $\frac{1}{2}g^2 V^2$ in the Wess-Zumino gauge. In fact, in this gauge, this function can be determined by noting:

$$\exp V_{WZ} = 1 - \theta\sigma^\mu\bar\theta A_\mu + i\theta\theta\bar\theta\bar\lambda - i\bar\theta\bar\theta\theta\lambda + \frac{1}{2}\,\theta\theta\bar\theta\bar\theta\,(D - \frac{1}{2}A^\mu A_\mu), \tag{22}$$

for an abelian Vector superfield. Here as usual λ denotes the gaugino field while A_μ represents the gauge field. D represents the auxiliary field of the Vector multiplet. The extension to the non-abelian case is straight forward.

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bosons with the Higgs fields and Higgs-Higgsino-gaugino vertices.

The second possible function of the superfields is the analytic or holomorphic function[11] of the superfields called the superpotential, W. This function essentially gives the interaction part of the lagrangian which is independent of the gauge couplings, like the Yukawa couplings. If renormalisability is demanded, the dimension of the superpotential is restricted to be less than or equal to three, $[W] \leq 3$ *i.e,* only products of three or less number of chiral superfields are allowed. Imposing this restriction of renormalisability the most general G_{SM} gauge invariant form of the W for the matter spectrum of MSSM (14,15) is given as

$$W = W_1 + W_2, \tag{23}$$

where

$$W_1 = h_{ij}^u Q_i U_j^c H_2 + h_{ij}^d Q_i D_j^c H_1 + h_{ij}^e L_i E_j^c H_1 + \mu H_1 H_2 \tag{24}$$

$$W_2 = \epsilon_i L_i H_2 + \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} U_i^c D_j^c D_k^c. \tag{25}$$

Here we have arranged the entire superpotential in to two parts, W_1 and W_2 with a purpose. Though both these parts are gauge invariant, W_2 also violates the global lepton number and baryon quantum numbers. The simultaneous presence of both these set of operators can lead to rapid proton decay and thus can make the MSSM phenomenologically invalid. For these reasons, one typically imposes an additional symmetry called R-parity in MSSM which removes all the dangerous operators in W_2. We will deal with R-parity in greater detail in the next section. For the present, let us just set W_2 to be zero due to a symmetry called R-parity and just call W_1 as W. The lagrangian can be derived from the superpotential containing (mostly) gauge invariant product of the three superfields by taking the $\theta\theta$ component, which can be represented in the integral form as

$$\mathcal{L}_{yuk} = \int d\theta^2 \ W(\Phi) + \int d\bar{\theta}^2 \ \bar{W}(\bar{\Phi}) \tag{26}$$

This part gives[12] the standard Yukawa couplings for the fermions with the Higgs, in addition also give the fermion-sfermion-higgsino couplings and scalar terms. For example, the coupling

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[11] This would mean that W is purely a function of complex fields $(z_1 z_2 z_3)$ or its conjugates $(z_1^\star z_2^\star z_3^\star)$.

[12] The $\theta\theta$ components of the product of three chiral superfields is given as[1]

$$\Phi_i \Phi_j \Phi_k |_{\theta\theta} = -\psi_i \psi_j \phi_k - \psi_j \psi_k \phi_i - \psi_k \psi_i \phi_j + F_i \phi_j \phi_k + F_j \phi_k \phi_i + F_k \phi_i \phi_j, \tag{27}$$

where as earlier, ψ_i represents the fermionic, ϕ_i the scalar and F_i the auxiliary component of the chiral superfield Φ_i. Similarly for the product of two superfields on has :

$$\Phi_i \Phi_j |_{\theta\theta} = -\psi_i \psi_j + F_i \phi_j + F_j \phi_i \tag{28}$$

$h_{ij}^u \; Q_i u_j^c H_2$ in the superpotential has the following expansion in terms of the component fields :

$$\mathcal{L}_{yuk} \supset h_{ij}^u \; Q_i u_j^c H_2 \mid_{\theta\theta}$$

$$\supset h_{ij}^u \; (\; u_i u_j^c H_2^0 - d_i u_j^c H_2^+ \;) \mid_{\theta\theta}$$

$$\supset h_{ij}^u (\psi_{u_i} \psi_{u_j^c} \phi_{H_2^0} + \phi_{\tilde{u}_i} \psi_{u_j^c} \psi_{\tilde{H}_2^0} + \psi_{u_i} \phi_{\tilde{u}_j^c} \psi_{\tilde{H}_2^0} - \psi_{d_i} \psi_{u_j^c} \phi_{H_2^+} - \phi_{\tilde{d}_i} \psi_{u_j^c} \psi_{\tilde{H}_2^+} - \psi_{d_i} \phi_{\tilde{u}_j^c} \psi_{\tilde{H}_2^+}) \tag{29}$$

$$\equiv h_{ij}^u \; (\; u_i u_j^c H_2^0 + \; \tilde{u}_i u_j^c \tilde{H}_2^0 + u_i \tilde{u}_j^c \tilde{H}_2^0 - d_i u_j^c H_2^+ \; - \tilde{d}_i u_j^c \tilde{H}_2^+ \; - d_i \tilde{u}_j^c \tilde{H}_2^+ \;), \tag{30}$$

where in the last equation, we have used the same notation for the chiral superfield as well as for its lowest component namely the scalar component. Note that we have not written the F-terms which give rise to the scalar terms in the potential. Similarly, there is the μ term which gives 'Majorana' type mass term for the Higgsino fields.

Finally, for every vector superfield (or a set of superfields) we have an associated field strength superfield $\mathcal{W}^\alpha$, which gives the kinetic terms for the gauginos and the field strength tensors for the gauge fields. Given that it is a chiral superfield, the component expansion is given by taking the $\theta\theta$ component of 'square' of that superfield[13]. In the MSSM, we have to add the corresponding field strength $\mathcal{W}$ superfields for electroweak vector superfields, W and B as well as for the gluonic G vector superfields of eqs.(20).

So far we have kept the auxiliary fields (D and F) of various chiral and vector superfields in the component form of our lagrangian. However, given that these fields are unphysical, they have to be removed from the lagrangian to go "on-shell". To eliminate the D and F fields, we have to use the equations of motions of these fields which have simple solutions for the F and D as :

$$F_i = \frac{\partial W}{\partial \phi_i} \quad ; \quad D_A = -g_A \; \phi_i^\star \; T_{ij}^A \; \phi_j, \tag{32}$$

where ϕ_i represents all the scalar fields present in MSSM. The index A runs over all the gauge groups in the model. For example, for $U(1)_Y$, $T_{ij}^A = (Y^2/2)\delta_{ij}$. The F and D terms together form the scalar potential of the MSSM[14] which is given as

$$V = \sum_i |F_i|^2 \; + \; \frac{1}{2} \; D^A D_A \tag{33}$$

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[13] In the Wess-Zumino gauge, $\mathcal{W}_\alpha = -\frac{1}{4}\bar{\mathcal{D}}\bar{\mathcal{D}}\mathcal{D}_\alpha V_{WZ}$ [1] ($\mathcal{D}$ is the differential operator on superfields) and the lagrangian has the form :

$$\mathcal{L} \supset \frac{1}{4} \left(\mathcal{W}^\alpha \mathcal{W}_\alpha|_{\theta\theta} + \mathcal{W}_{\dot{\alpha}} \mathcal{W}^{\dot{\alpha}}|_{\bar{\theta}\bar{\theta}} \right) = \frac{1}{2}D^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - i\lambda\sigma^\mu\partial_\mu\bar{\lambda} \tag{31}$$

D represents the auxiliary component of the vector superfields. The extension to non-abelian vector superfields in straight forward.

[14] Later we will see that there are also additional terms which contribute to the scalar potential which come from the supersymmetry breaking sector.

Putting together, we see that the lagrangian of the MSSM with SUSY unbroken is of the form :

$$\mathcal{L}_{MSSM}^{(0)} = \int \left(d\theta^2 \ W(\Phi) + H.c \right) + \int d\theta^2 \ d\bar{\theta}^2 \ \Phi_i^\dagger \ e^{gV} \ \Phi_i + \int \left(d\theta^2 \ \mathcal{W}^\alpha \mathcal{W}_\alpha + H.c \right). \qquad (34)$$

where all the functions appearing in (34) have been discussed in eqs.(21,24) and (31).

2. *Think it over*

- The full supersymmetric lagrangian of the Standard Model can be constructed from the prescription given in the above section. Identify the dominant one-loop contributions for the Higgs particle. Note that SUSY is still unbroken. What are the dominant 1-loop contributions for other scalar particles, say the stop ? Compute the processes $\mu \ \rightarrow \ e + \ \gamma$ and $K^0 - \bar{K}^0$ mixing in this limit.

- As we have seen, W is a holomorphic function and that there are two Higgs doublets giving masses to up type and down type quarks separately. (a) Give examples of operators which are gauge invariant but non-holomorphic ? (b) Show that such operators involving the Higgs fields will lead to Yukawa like couplings with the "wrong" Higgs. Study the implications of such couplings.

Historical Note

Supersymmetries were first introduced in the context of string theories by Ramond. In quantum field theories, this symmetry is realised through fermionic generators, thus escaping the no-go theorems of Coleman and Mandula [17]. The simplest Lagrangian realising this symmetry in four dimensions was built by Wess and Zumino which contains a spin $\frac{1}{2}$ fermion and a scalar.

In particle physics, supersymmetry plays an important role in protecting the Higgs mass. To understand how it protects the Higgs mass, let us consider the hierarchy problem once again. The Higgs mass enters as a bare mass parameter in the Standard Model lagrangian, eq.(10). Contributions from the self energy diagrams of the Higgs are quadratically divergent pushing the Higgs mass up to cut-off scale. In the absence of any new physics at the intermediate energies, the cut-off scale is typically M_{GUT} or M_{planck}. Cancellation of these divergences with the bare mass parameter would require fine-tuning of order one part in 10^{-36} rendering the theory 'unnatural'[18]. In a complete GUT model like SU(5) this might reflect as a severe problem of doublet-triplet splitting [19, 20]. On the other hand, if one has additional contributions, say, for example, for the diagram with the Higgs self coupling, there is an additional contribution from a fermionic loop, with

the fermion carrying the same mass as the scalar, the contribution from this additional diagram would now cancel the quadratically divergent part of the SM diagram, with the total contribution now being only logarithmically divergent. If this mechanism needs to work for all the diagrams, not just for the Higgs self-coupling and for all orders in perturbation theory, it would require a symmetry which would relate a fermion and a boson with same mass. Supersymmetry is such a symmetry.

A. R-parity

In the previous section, we have seen that there are terms in the superpotential, eq.(25) which are invariant under the Standard Model gauge group G_{SM} but however violate baryon (B) and individual lepton numbers ($L_{e,\mu,\tau}$). At the first sight, it is bit surprising : the matter *superfields* carry the same quantum numbers under the G_{SM} just like the ordinary matter fields do in the Standard Model and B and $L_{e,\mu,\tau}$ violating terms are not present in the Standard Model. The reason can be traced to the fact that in the MSSM, where matter sector is represented in terms of superfields, there is no distinction between the fermions and the bosons of the model. In the Standard Model, the Higgs field is a boson and the leptons and quarks are fermions and they are different representations of the Lorentz group. This distinction is lost in the MSSM, the Higgs superfield, H_1 and the lepton superfields L_i have the same quantum numbers under G_{SM} and given that they are both (chiral) superfields, there is no way of distinguishing them. For this reason, the second part of the superpotential W_2 makes an appearance in supersymmetric version of the Standard Model. In fact, the first three terms of eq.(25) can be achieved by replacing $H_1 \rightarrow L_i$ in the terms containing H_1 of W_1.

The first three terms of the second part of the superpotential W_2 (eq.(25)), are lepton number violating whereas the last term is baryon number violating. The simultaneous presence of both these interactions can lead to proton decay, for example, through a squark exchange. An example of such an process in given in Figure 1. Experimentally the proton is quite stable. In fact its life time is pretty large $\gtrsim \mathcal{O}(10^{33})$ years [21]. Thus products of these couplings (λ'' and one of (λ', ϵ, λ) which can lead to proton decay are severely constrained to be of the order of $(O)(10^{-20})$[15]. Thus to make the MSSM phenomenologically viable one should expect these couplings in W_2 to take

---

[15] The magnitude of these constraints depends also on the scale of supersymmetry breaking, which we will come to discuss only in the next section. For a list of constraints on R-violating couplings, please see G. Bhattacharyya [22].
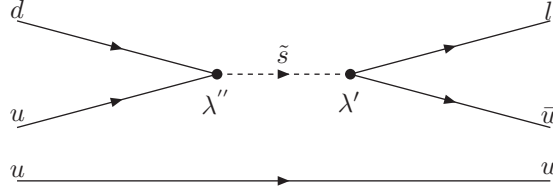
FIG. 1: A sample diagram showing the decay of the proton in the presence of R-parity violating couplings.

such extremely small values.

A more natural way of dealing with such small numbers for these couplings would be to set them to be zero. This can be arrived at by imposing a discrete symmetry on the lagrangian called R-parity. R-parity has been originally introduced as a discrete R-symmetry [16] by Ferrar and Fayet [23] and then later realised to be of the following form by Ferrar and Weinberg [24] acting on the component fields:

$$R_p = (-1)^{3(B-L)+2s}, \tag{35}$$

where B and L represent the Baryon and Lepton number respectively and s represents the spin of the particle. Under R-parity the transformation properties of various superfields can be summarised as:

$$
\begin{aligned}
\{V_s^A, V_w^I, V_y\} &\rightarrow \{V_s^A, V_w^I, V_y\} \\
\theta &\rightarrow -\theta^\star \\
\{Q_i, U_i^c, D_i^c, L_i, E_i^c\} &\rightarrow -\{Q_i, U_i^c, D_i^c, L_i, E_i^c\} \\
\{H_1, H_2\} &\rightarrow \{H_1, H_2\}
\end{aligned}
\tag{36}
$$

---

[16] R-symmetries are symmetries under which the θ parameter transform non-trivially.

Imposing these constraints on the superfields will now set all the couplings in W_2 to zero.

Imposing R-parity has an advantage that it provides a natural candidate for dark matter. This can be seen by observing that R-parity distinguishes a particle from its superpartner. This ensures that every interaction vertex has at least two supersymmetric partners when R-parity is conserved. The lightest supersymmetric particle (LSP) cannot decay in to a pair of SM particles and remains stable. R-parity can also be thought of as a remnant symmetry theories with an additional $U(1)$ symmetry, which is natural in a large class of supersymmetric Grand Unified theories. Finally, one curious fact about R-parity : it should be noted that R-parity constraints baryon and lepton number violating couplings of dimension four or rather only at the renormalisable level. If one allows for non-renormalisable operators in the MSSM, *i.e* that is terms of dimension more than three in the superpotential, they can induce dim 6 operators which violate baryon and lepton numbers at the lagrangian level and are still allowed by R-parity. Such operators are typically suppressed by high mass scale $\sim M_{Pl}$ or M_{GUT} and thus are less dangerous. In the present set of lectures, we will always impose R-parity in the MSSM so that the proton does not decay, though there are alternatives to R-parity which can also make proton stable.

1. *Think it over*

- Is imposing R-parity the only way to get rid of the terms which lead to proton decay ? (Hint: For proton decay to occur both L and B violating operators are required. R-parity removes both these sets of operators which is unnecessary. We can think of discrete symmetries which can remove only either B or L type of operators.) See for example[25].

V. STEP 3: SUPERSYMMETRY BREAKING

So far, we have seen that the Supersymmetric Standard Model lagrangian can also be organised in a similar way like the Standard Model lagrangian though one uses functions of superfields now to get the lagrangian rather than the ordinary fields. In the present section we will cover the last part (term) of the total MSSM lagrangian

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{gauge/kinetic}}\left(K(\Phi, V)\right) + \mathcal{L}_{\text{yukawa}}\left(W(\Phi)\right) + \mathcal{L}_{\text{scalar}}\left(F^2, D^2\right) + \mathcal{L}_{\text{SSB}} \qquad (37)$$

which we have left out so far and that concerns supersymmetry breaking (SSB). Note that the first three terms are essentially from $\mathcal{L}_{\text{MSSM}}^{(0)}$ of eq.(34). In Nature, we do not observe supersymmetry. Supersymmetry breaking has to be incorporated in the MSSM to make it realistic. In a general
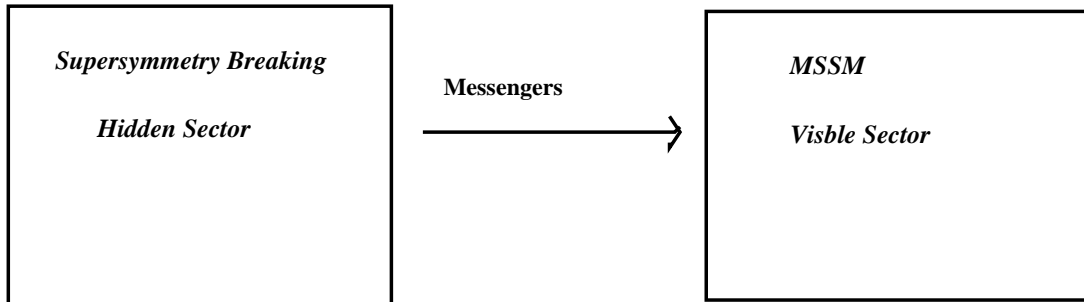
FIG. 2: A schematic diagram showing SUSY breaking using Hidden sector models

lagrangian, supersymmetry can be broken spontaneously if the auxiliary fields F or D appearing in the definitions of the chiral and vector superfields respectively attain a vacuum expectation value (*vev*). If the F fields get a *vev*, it is called F-breaking whereas if the D fields get a *vev*, it is called D-breaking.

Incorporation of spontaneous SUSY breaking in MSSM would mean that at least one (or more) of the F-components corresponding to one (or more) of the MSSM chiral (matter) superfields would attain a vacuum expectation value. However, this approach fails as this leads to phenomenologically unacceptable prediction that at least one of the super-partner should be lighter (in mass) than the ordinary particle. This is not valid phenomenologically as such a light super partner (of SM particle) has been ruled out experimentally. One has to think of a different approach for incorporating supersymmetry breaking in to the MSSM [26].

One of the most popular and successful approaches has been to assume another sector of the theory consisting of superfields which are not charged under the Standard Model gauge group. Such a sector of the theory is called 'Hidden Sector' as they cannot been "seen" like the Standard Model particles and remain hidden. Supersymmetry can be broken spontaneously in this sector. This information is communicated to the visible sector or MSSM through a messenger sector. The messenger sector can be made up of gravitational interactions or ordinary gauge interactions. The communication of supersymmetry breaking leads to supersymmetry breaking terms in MSSM. Thus, supersymmetry is not broken spontaneously within the MSSM, but *explicitly* by adding supersymmetry breaking terms in the lagrangian.

However, not all supersymmetric terms can be added. We need to add only those terms which do not re-introduce quadratic divergences back into the theory[17]. It should be noted that in most

---

[17] Interaction terms and other couplings which do not lead to quadratically divergent (in cut-off Λ) terms in the theory once loop corrections are taken in to consideration. It essentially means we only add dimensional full couplings which are supersymmetry breaking.

models of spontaneous supersymmetry breaking, only such terms are generated. These terms which are called "soft" supersymmetry breaking terms can be classified as follows:

- a) Mass terms for the gauginos which are a part of the various vector superfields of the MSSM.

- b) Mass terms for the scalar particles, $m^2_{\phi_{ij}} \phi^\star_i \phi_j$ with $\phi_{i,j}$ representing the scalar partners of chiral superfields of the MSSM.

- c) Trilinear scalar interactions, $A_{ijk}\phi_i\phi_j\phi_k$ corresponding to the cubic terms in the superpotential.

- d) Bilinear scalar interactions, $B_{ij}\phi_i\phi_j$ corresponding to the bilinear terms in the superpotential.

Note that all the above terms are dimensionful. Adding these terms would make the MSSM non-supersymmetric and thus realistic. The total MSSM lagrangian is thus equal to

$$\mathcal{L}_{total} = \mathcal{L}^{(0)}_{MSSM} + \mathcal{L}_{SSB} \tag{38}$$

with $\mathcal{L}^{(0)}_{MSSM}$ given in eq.(34). Sometimes in literature we have $\mathcal{L}_{SSB} = \mathcal{L}_{soft}$. Let us now see the complete list of all the soft SUSY breaking terms one can incorporate in the MSSM:

1. *Gaugino Mass terms*: Corresponding to the three vector superfields (for gauge groups $U(1)$, $SU(2)$ and $SU(3)$) we have $\tilde{B}, \tilde{W}$ and $\tilde{G}$) we have three gaugino mass terms which are given as $M_1\tilde{B}\tilde{B}$, $M_2\tilde{W}_I\tilde{W}_I$ and $M_3\tilde{G}_A\tilde{G}_A$, where $I(A)$ runs over all the $SU(2)(SU(3))$ group generators.

2. *Scalar Mass terms*: For every scalar in each chiral (matter) superfield , we can add a mass term of the form $m^2 \phi^\star_i \phi_j$. Note that the generation indices i,j *need* not be the same. Thus the mass terms can violate flavour. Further, given that SUSY is broken prior to $SU(2) \times U(1)$ breaking , all these mass terms for the scalar fields should be written in terms of their 'unbroken' $SU(2) \times U(1)$ representations. Thus the scalar mass terms are : $m^2_{Q_{ij}}\tilde{Q}^\dagger_i\tilde{Q}_j$, $m^2_{u_{ij}}\tilde{u}^c{}^\star_i\tilde{u}^c{}_j$, $m^2_{d_{ij}}\tilde{d}^c{}^\star_i\tilde{d}^c{}_j$, $m^2_{L_{ij}}\tilde{L}^\dagger_i\tilde{L}_j$, $m^2_{e_{ij}}\tilde{e}^c{}^\star_i\tilde{e}^c{}_j$, $m^2_{H_1}H^\dagger_1H_1$ and $m^2_{H_2}H^\dagger_2H_2$.

3. *Trilinear Scalar Couplings*: As mentioned again, there are only three types of trilinear scalar couplings one can write which are G_{SM} gauge invariant. In fact, their form exactly follows from the Yukawa couplings. These are : $A^u_{ij}\tilde{Q}_i\tilde{u}^c_jH_2$, $A^d_{ij}\tilde{Q}_i\tilde{d}^c_jH_1$ and $A^e_{ij}\tilde{L}_i\tilde{e}^c_jH_1$.

4. *Bilinear Scalar Couplings*: Finally, there is only one bilinear scalar coupling (other than the mass terms) which is gauge invariant. The form of this term also follows from the superpotential. It is given as : BH_1H_2.

Adding all these terms completes the lagrangian for the MSSM. However, the particles are still not in their 'physical' basis as $SU(2) \times U(1)$ breaking is not yet incorporated. Once incorporated the physical states of the MSSM and their couplings could be derived.

VI. STEP 4: $SU(2) \times U(1)$ BREAKING

As a starting point, it is important to realize that the MSSM is a two Higgs doublet model *i.e*, SM with two Higgs doublets instead of one, with a different set of couplings [27]. Just as in Standard Model, spontaneous breaking of $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$ can be incorporated here too. Doing this leads to constraints relating various parameters of the model. To see this, consider the neutral Higgs part of the total scalar potential including the soft terms. It is given as

$$
\begin{aligned}
V_{scalar} \; = \; & (m_{H_1}^2 + \mu^2)|H_1^0|^2 + (m_{H_2}^2 + \mu^2)|H_2^0|^2 - (B_\mu \mu H_1^0 H_2^0 + H.c) \\
& + \; \frac{1}{8}(g^2 + g'^2)(H_2^{0\,2} - H_1^{0\,2})^2 + \dots,
\end{aligned}
\tag{39}
$$

where H_1^0, H_2^0 stand for the neutral Higgs scalars and we have parameterised the bilinear soft term $B \equiv B_\mu \mu$. Firstly, we should require that the potential should be bounded from below. This gives the condition (in field configurations where the D-term goes to zero, *i.e*, the second line in eq.(39)):

$$
2B_\mu < 2|\mu|^2 \; + \; m_{H_2}^2 \; + m_{H_1}^2
\tag{40}
$$

Secondly, the existence of a minima for the above potential would require at least one of the Higgs mass squared to be negative giving the condition, (determinant of the 2×2 neutral Higgs mass squared matrix should be negative)

$$
B_\mu^2 > (|\mu|^2 \; + \; m_{H_2}^2 \;) \; (|\mu|^2 + m_{H_1}^2)
\tag{41}
$$

In addition to ensuring the existence of a minima, one would also require that the minima should be able to reproduce the standard model relations i.e, correct gauge boson masses. We insist that both the neutral Higgs attain vacuum expectation values :

$$
< H_1^0 > = \frac{v_1}{\sqrt{2}} \quad ; \quad < H_2^0 > = \frac{v_2}{\sqrt{2}}
\tag{42}
$$

18

and furthermore we define

$$v_1^2 + v_2^2 \;=\; v^2 \;=\; 246^2 \text{ GeV}^2,$$

where v represents the vev of the Standard Model (SM) Higgs field. However, these *vev*s should correspond to the minima of the MSSM potential. The minima are derived by requiring $\partial V/\partial H_1^0 = 0$ and $\partial V/H_2^0 = 0$ at the minimum, where the form of V is given in eq.(39). These derivative conditions lead to relations between the various parameters of the model at the minimum of the potential. We have, using the Higgs vev (42) and the formulae for[18] $M_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2$, the minimisation conditions can rewritten as

$$
\begin{aligned}
\frac{1}{2} M_Z^2 &= \frac{m_{H_1}^2 - \tan^2\beta\ m_{H_2}^2}{\tan^2\beta - 1} - \mu^2 \\
\text{Sin}2\beta &= \frac{2 B_\mu\ \mu}{m_{H_2}^2 + m_{H_1}^2 + 2\mu^2},
\end{aligned}
\tag{43}
$$

where we have used the definition $\tan\beta = v_2/v_1$ as the ratio of the vacuum expectation values of H_2^0 and H_1^0 respectively. Note that the parameters $m_{H_1}^2$, $m_{H_2}^2$, B_μ are all supersymmetry breaking 'soft' terms. μ is the coupling which comes in the superpotential giving the supersymmetry conserving masses to the Higgs scalars. These are related to the Standard Model parameters M_Z and a ratio of *vev*s, parameterised by an angle $\tan\beta$. Thus these conditions relate SUSY breaking soft parameters with the SUSY conserving ones and the Standard Model parameters. For any model of supersymmetry to make contact with reality, the above two conditions (43)need to be satisfied.

The above minimisation conditions are given for the 'tree level' potential only. 1-loop corrections a 'la Coleman-Weinberg can significantly modify these minima. We will discuss a part of them in later sections when we discuss the Higgs spectrum. Finally we should mention that, in a more concrete approach, one should consider the entire scalar potential including all the scalars in the theory, not just confining ourselves to the neutral Higgs scalars. For such a potential one should further demand that there are no deeper minima which are color and charge breaking (which effectively means none of the colored and charged scalar fields get vacuum expectation values). These conditions lead to additional constraints on parameters of the MSSM[28].

---

[18] In this lecture note, we will be using $g_2 = g = g_W$ for the SU(2) coupling, whereas $g' = g_1$ for the $U(1)_Y$ coupling and $g_s = g_3$ for the SU(3) strong coupling.

2. Think it over

- In the MSSM, we have considered here contains two Higgs doublets. In addition to H_1 and H_2, consider an additional Higgs field field S, which transforms as a singlet under all the gauge groups of G_{SM}. Write down the superpotential including the singlet field S invariant under G_{SM}. Derive the corresponding scalar potential including the soft SUSY breaking terms. Minimise the neutral Higgs potential and derive the electro-weak minimisation conditions. How many are there and what are they? (Hint: Assume the S field also develops a *vev* and that its *vev* is much larger than v_1 and v_2.)

VII.   STEP 5: MASS SPECTRUM

We have seen in the earlier section, supersymmetry breaking terms introduce mass-splittings between ordinary particles and their super-partners. Given that particles have zero masses in the limit of exact G_{SM}, only superpartners are given soft mass terms. After the $SU(2) \times U(1)$ breaking, ordinary particles as well as superparticles attain mass terms. For the supersymmetric partners, these mass terms are either additional contributions or mixing terms between the various super-particles. Thus, just like in the case of ordinary SM fermions, where one has to diagonalise the fermion mass matrices to write the lagrangian in the 'on-shell' format or the physical basis, a similar diagonalisation has to be done for the super-symmetric particles and their mass matrices.

3. The Neutralino Sector

To begin with lets start with the gauge sector. The superpartners of the neutral gauge bosons (neutral gauginos) and the fermionic partners of the neutral higgs bosons (neutral higgsinos) mix to form Neutralinos. The neutralino mass matrix in the basis

$$\mathcal{L} \supset \frac{1}{2} \Psi_N \mathcal{M}_N \Psi_N^T + H.c$$

where $\Psi_N = \{\tilde{B}, \ \tilde{W}^0, \tilde{H}_1^0, \tilde{H}_2^0\}$ is given as :

$$\mathcal{M}_N = \begin{pmatrix} M_1 & 0 & -M_Z c\beta \ s\theta_W & M_Z s\beta \ s\theta_W \\ 0 & M_2 & M_Z c\beta \ c\theta_W & M_Z s\beta \ c\theta_W \\ -M_Z c\beta \ s\theta_W & M_Z c\beta \ c\theta_W & 0 & -\mu \\ M_Z s\beta \ s\theta_W & -M_Z s\beta \ c\theta_W & -\mu & 0 \end{pmatrix}, \tag{44}$$

with $c\beta(s\beta)$ and $c\theta_W(s\theta_W)$ standing for $\cos\beta(\sin\beta)$ and $\cos\theta_W(\sin\theta_W)$ respectively. As mentioned earlier, M_1 and M_2 are the soft parameters, whereas μ is the superpotential parameter, thus SUSY conserving. The angle β is typically taken as a input parameter, $tan\beta = v_2/v_1$ whereas θ_W is the Weinberg angle given by the inverse tangent of the ratio of the gauge couplings as in the SM. Note that the neutralino mass matrix being a Majorana mass matrix is complex symmetric in nature. Hence it is diagonalised by a unitary matrix N,

$$N^* \cdot M_{\tilde{N}} \cdot N^\dagger = \text{Diag.}(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0}) \tag{45}$$

The states are rotated by $\chi_i^0 = N^\star \Psi$ to go the physical basis.

4. The Chargino Sector

In a similar manner to the neutralino sector, all the fermionic partners of the charged gauge bosons and of the charged Higgs bosons mix after electroweak symmetry breaking. However, they combine in a such a way that a Wino-Higgsino Weyl fermion pair forms a Dirac fermion called the chargino. This mass matrix is given as

$$\mathcal{L} \supset -\frac{1}{2}\,(\,\tilde{W}^-\quad \tilde{H}_1^-\,)\begin{pmatrix} M_2 & \sqrt{2}M_W\sin\beta \\ \sqrt{2}M_W\cos\beta & \mu \end{pmatrix}\begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_2^+ \end{pmatrix}, \tag{46}$$

Given the non-symmetric (non-hermitian) matrix nature of this matrix, it is diagonalised by a bi-unitary transformation, $U^* \cdot M_C \cdot V^\dagger = \text{Diag.}(m_{\chi_1^+}, m_{\chi_2^+})$. The chargino eigenstates are typically represented by $\chi^\pm$ with mass eigenvalues $m_{\chi^\pm}$. The explicit forms for U and V can be found by the eigenvectors of $M_C M_C^\dagger$ and $M_C^\dagger M_C$ respectively [29].

5. The Sfermion Sector

Next let us come to the sfermion sector. Remember that we have added different scalar fields for the right and left handed fermions in the Standard Model. After electroweak symmetry breaking, the sfermions corresponding to the left fermion and the right fermion mix with each other. Furthermore while writing down the mass matrix for the sfermions, we should remember that these terms could break the flavour *i.e*, we can have mass terms which mix different generation. Thus, in general the sfermion mass matrix is a 6×6 mass matrix given as :

$$\xi^\dagger\, M_{\tilde{f}}^2 \xi \;\; ; \quad \xi = \{\tilde{f}_{L_i}, \tilde{f}_{R_i}\}$$

From the total scalar potential, the mass matrix for these sfermions can be derived using standard definition given as

$$
m_{ij}^2 = \begin{pmatrix} \frac{\partial^2 V}{\partial \phi_i \partial \phi_j^\star} & \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \\ \frac{\partial^2 V}{\partial \phi_i^\star \partial \phi_j^\star} & \frac{\partial^2 V}{\partial \phi_i^\star \partial \phi_j} \end{pmatrix}
\tag{47}
$$

Using this for sfermions, we have :

$$
M_{\tilde{f}}^2 = \begin{pmatrix} m_{\tilde{f}_{\mathrm{LL}}}^2 & m_{\tilde{f}_{\mathrm{LR}}}^2 \\ m_{\tilde{f}_{\mathrm{LR}}}^{2\,\dagger} & m_{\tilde{f}_{\mathrm{RR}}}^2 \end{pmatrix},
\tag{48}
$$

where each of the above entries represents 3×3 matrices in the generation space. More specifically, they have the form (as usual, i, j are generation indices):

$$
\begin{aligned}
m_{\tilde{f}_{L_i L_j}}^2 &= M_{\tilde{f}_{L_i L_j}}^2 + m_f^2 \delta_{ij} + M_Z^2 \cos 2\beta (T_3 + \sin^2 \theta_W Q_{\mathrm{em}}) \delta_{ij} \\
m_{\tilde{f}_{L_i R_j}}^2 &= \left((Y_f^A \cdot \substack{v_2 \\ v_1} - m_f \mu \substack{\tan\beta \\ \cot\beta}) \ \text{for} \ f = \substack{e,d \\ u} \right) \delta_{ij} \\
m_{\tilde{f}_{\mathrm{RR}}}^2 &= M_{\tilde{f}_{R_{ij}}}^2 + \left(m_f^2 + M_Z^2 \cos 2\beta \sin^2 \theta_W Q_{\mathrm{em}} \right) \delta_{ij}
\end{aligned}
\tag{49}
$$

In the above, $M_{\tilde{f}_L}^2$ represents the soft mass term for the corresponding fermion (L for left, R for right), T_3 is the eigenvalue of the diagonal generator of $SU(2)$, m_f is the mass of the fermion with Y and Q_{em} representing the hypercharge and electromagnetic charge (in units of the charge of the electron) respectively. The sfermion mass matrices are hermitian and are thus diagonalised by a unitary rotation, $R_{\tilde{f}} R_{\tilde{f}}^\dagger = 1$:

$$
R_{\tilde{f}} \cdot M_{\tilde{f}} \cdot R_{\tilde{f}}^\dagger = \mathrm{Diag.}(m_{\tilde{f}_1}, m_{\tilde{f}_2}, \ldots, m_{\tilde{f}_6})
\tag{50}
$$

6. *The Higgs sector*

Now let us turn our attention to the Higgs fields. We will use again use the standard formula of eq.(47), to derive the Higgs mass matrices. The eight Higgs degrees of freedom form a 8×8 Higgs mass matrix which breaks down diagonally in to three 2×2 mass matrices[19].

The mass matrices are divided in to charged sector, CP odd neutral and CP even neutral. This helps us in identifying the goldstone modes and the physical spectrum in an simple manner. Before writing down the mass matrices, let us first define the following parameters :

$$
m_1^2 = m_{H_1}^2 + \mu^2, \quad m_2^2 = m_{H_2}^2 + \mu^2, \quad m_3^2 = B_\mu \mu.
$$

---

[19] The discussion in this section closely follows from the discussion presented in Ref.[30]

In terms of these parameters, the various mass matrices and the corresponding physical states obtained after diagonalising the mass matrices are given below:

Charged Higgs and Goldstone Modes:

$$\left(H_1^+ \ H_2^+ \right) \begin{pmatrix} m_1^2 + \frac{1}{8}(g_1^2 + g_2^2)(v_1^2 - v_2^2) + \frac{1}{4}g_2^2 v_2^2 & m_3^2 + \frac{1}{4}g_2^2 v_1 v_2 \\ m_3^2 + \frac{1}{4}g_2^2 v_1 v_2 & m_2^2 - \frac{1}{8}(g_1^2 + g_2^2)(v_1^2 - v_2^2) + \frac{1}{4}g_2^2 v_2^2 \end{pmatrix} \begin{pmatrix} H_1^- \\ H_2^- \end{pmatrix}$$

(51)

Using the minimisation conditions (43), this matrix becomes,

$$\left(H_1^+ \ H_2^+ \right) \left(\frac{m_3^2}{v_1 v_2} + \frac{1}{4}g_2^2 \right) \begin{pmatrix} v_2^2 & v_1 v_2 \\ v_1 v_2 & v_1^2 \end{pmatrix} \begin{pmatrix} H_1^- \\ H_2^- \end{pmatrix}$$

(52)

which has determinant zero leading to the two eigenvalues as :

$$m_{G^\pm}^2 = 0$$

$$m_{H^\pm}^2 = \left(\frac{m_3^2}{v_1 v_2} + \frac{1}{4}g_2^2 \right)(v_1^2 + v_2^2),$$

(53)

$$= \frac{2m_3^2}{sin2\beta} + M_W^2$$

(54)

where $G^\pm$ represents the Goldstone mode. The physical states are obtained just by rotating the original states in terms of the H_1, H_2 fields by an mixing angle. The mixing angle in the present case (in the unitary gauge) is just $tan\beta$:

$$\begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix} = \begin{pmatrix} sin\beta & cos\beta \\ -cos\beta & sin\beta \end{pmatrix} \begin{pmatrix} H^\pm \\ G^\pm \end{pmatrix}$$

(55)

CP odd Higgs and Goldstone Modes:

Let us now turn our attention to the CP-odd Higgs sector. The mass matrices can be written in a similar manner but this time for imaginary components of the neutral Higgs.

$$\left(ImH_1^0 \ ImH_2^0 \right) \begin{pmatrix} m_1^2 + \frac{1}{8}(g_1^2 + g_2^2)(v_1^2 - v_2^2) & m_3^2 \\ m_3^2 & m_2^2 - \frac{1}{8}(g_1^2 + g_2^2)(v_1^2 - v_2^2) \end{pmatrix} \begin{pmatrix} ImH_1^0 \\ ImH_2^0 \end{pmatrix}$$

(56)

As before, again using the minimisation conditions, this matrix becomes,

$$\left(ImH_1^0 \ ImH_2^0 \right) m_3^2 \begin{pmatrix} v_2/v_1 & 1 \\ 1 & v_1/v_2 \end{pmatrix} \begin{pmatrix} ImH_1^0 \\ ImH_2^0 \end{pmatrix}$$

(57)

which has determinant zero leading to the two eigenvalues as :

$$m_{G^0}^2 = 0$$

$$m_{A^0}^2 = \left(\frac{m_3^2}{v_1 v_2} \right)(v_1^2 + v_2^2) = \frac{2m_3^2}{sin2\beta}$$

(58)

Similar to the charged sector, the mixing angle between these two states in the unitary gauge is again just tanβ.

$$\frac{1}{\sqrt{2}} \begin{pmatrix} A^0 \\ G^0 \end{pmatrix} = \begin{pmatrix} sin\beta & cos\beta \\ -cos\beta & sin\beta \end{pmatrix} \begin{pmatrix} ImH_1^0 \\ ImH_2^0 \end{pmatrix} \tag{59}$$

CP even Higgs:

Finally, let us come to the real part of the neutral Higgs sector. The mass matrix in this case is given by the following.

$$\begin{pmatrix} ReH_1^0 & ReH_2^0 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 2m_1^2 + \frac{1}{4}(g_1^2 + g_2^2)(3v_1^2 - v_2^2) & -2m_3^2 - \frac{1}{4}v_1v_2(g_1^2 + g_2^2) \\ -2m_3^2 - \frac{1}{4}v_1v_2(g_1^2 + g_2^2) & 2m_2^2 + \frac{1}{4}(g_1^2 + g_2^2)(3v_2^2 - v_1^2) \end{pmatrix} \begin{pmatrix} ReH_1^0 \\ ReH_2^0 \end{pmatrix} \tag{60}$$

Note that in the present case, there is no Goldstone mode. As before, we will use the minimisation conditions and further using the definition of m_A^2 from eq.(58), we have :

$$\begin{pmatrix} ReH_1^0 & ReH_2^0 \end{pmatrix} \begin{pmatrix} m_A^2 sin^2\beta + M_z^2 cos\beta & -(m_A^2 + m_Z^2)sin\beta cos\beta \\ -(m_A^2 + m_Z^2)sin\beta cos\beta & m_A^2 cos^2\beta + M_z^2 sin\beta \end{pmatrix} \begin{pmatrix} ReH_1^0 \\ ReH_2^0 \end{pmatrix} \tag{61}$$

The matrix has two eigenvalues which are given by the two signs of the following equation:

$$m_{H,h}^2 = \frac{1}{2} \left[m_A^2 + m_Z^2 \pm \{(m_A^2 + m_Z^2)^2 - 4m_Z^2 m_A^2 cos^2 2\beta\}^{1/2} \right] \tag{62}$$

The heavier eigenvalue m_H^2, is obtained by taken the positive sign, whereas the lighter eigenvalue m_h^2 is obtained by taking the negative sign respectively. The mixing angle between these two states can be read out from the mass matrix of the above[20] as :

$$tan\,2\alpha = \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2} \, tan\,2\beta \tag{63}$$

Tree Level Catastrophe:

So far we have seen that out of the eight Higgs degrees of freedom, three of them form the Goldstone modes after incorporating $SU(2) \times U(1)$ breaking and there are five *physical* Higgs bosons fields in the MSSM spectrum. These are the charged Higgs ($H^\pm$) a CP-odd Higgs (A) and two CP-even Higgs bosons (h, H). From the mass spectrum analysis above, we have seen that the mass

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[20] The mixing angle for a 2×2 symmetric matrix, C_{ij} is given by

$$tan2\theta = 2C_{12}/(C_{22} - C_{11}).$$

eigenvalues of these Higgs bosons are related to each other. In fact, putting together all the eigenvalue equations, we summarise the relations between them as follows :

$$\begin{aligned}
m_{H^\pm}^2 &= m_A^2 + m_W^2 > \max(M_W^2, m_A^2) \\
m_h^2 + m_H^2 &= m_A^2 + m_Z^2 \\
m_H &> max(m_A, m_Z) \\
m_h &< min(m_A, M_Z)|cos2\beta| < min(m_A, m_Z)
\end{aligned} \tag{64}$$

Let us concentrate on the last relation of the above eq.(64). The condition on the lightest CP even Higgs mass, m_h, tell us that it should be equal to m_Z in the limit $\tan\beta$ is saturated to be maximum, such that $\cos2\beta \to 1$ and $m_A \to \infty$. If these limits are not saturated, it is evident that the light higgs mass is less that m_Z. This is one of main predictions of MSSM which could make it easily falsifiable from the current generation of experiments like LEP, Tevatron and the upcoming LHC. Given that present day experiments have not found a Higgs less that Z-boson mass, it is tempting to conclude that the MSSM is not realised in Nature. However caution should be exercised before taking such a route as our results are valid only at the tree level. In fact, in a series of papers in the early nineties [31], it has been shown that large one-loop corrections to the Higgs mass can easily circumvent this limit.

The light Higgs Spectrum at 1-loop

As mentioned previously, radiative corrections can significantly modify the mass relations which we have presented in the previous section. As is evident, these corrections can be very important for the light Higgs boson mass. Along with the 1-loop corrections previously, in the recent years dominant parts of two-loop corrections have also been available [32] with a more complete version recently given[33]. In the following we will present the one-loop corrections to the light Higgs mass and try to understand the implications for the condition eq.(64). Writing down the 1-loop corrections to the CP-even part of the Higgs mass matrix as :

$$M_{Re}^2 = M_{Re}^2(0) + \delta M_{Re}^2, \tag{65}$$

where $M_{Re}^2(0)$ represents the tree level mass matrix given by eq.(61) and δM_{Re}^2 represents its one-loop correction. The dominant one-loop correction comes from the top quark and stop squark loops which can be written in the following form:

$$\delta M_{Re}^2 = \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{12} & \Delta_{22} \end{pmatrix}, \tag{66}$$

where

$$\Delta_{11} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 sin^2\beta}\left[\frac{\mu(A_t+\mu cot\beta)}{m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2}\right]^2\left(2-\frac{m_{\tilde{t}_1}^2+m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2}ln\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right)$$

$$\Delta_{12} = \frac{3G_F m_t^4}{2\sqrt{2}\pi^2 sin^2\beta}\left[\frac{\mu(A_t+\mu cot\beta)}{m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2}\right]ln\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}+\frac{A_t}{\mu}\Delta_{11}$$

$$\Delta_{22} = \frac{3G_F m_t^4}{\sqrt{2}\pi^2 sin^2\beta}\left[ln\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^2}+\frac{A_t(A_t+\mu cot\beta)}{m_{\tilde{t}_1}^2-m_{\tilde{t}_2}}ln\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right]+\frac{A_t}{\mu}\Delta_{11} \qquad (67)$$

In the above G_F represents Fermi Decay constant, m_t, the top mass, $m_{\tilde{t}_1}^2$, $m_{\tilde{t}_1}^2$ are the eigenvalues of the stop mass matrix and A_t is the trilinear scalar coupling (corresponding to the top Yukawa coupling) in the stop mass matrix. μ and the angle β have their usual meanings. Taking in to account these corrections, the condition (64) takes the form:

$$m_h^2 < m_Z^2 cos^2 2\beta + \Delta_{11} cos^2\beta + \Delta_{12} sin2\beta + \Delta_{22} sin^2\beta \qquad (68)$$

Given that m_t is quite large, almost twice the m_Z mass, for suitable values of the stop masses, it is clear that the tree level upper limit on the light Higgs mass is now evaded. However, a reasonable upper limit can still be got by assuming reasonable values for the stop mass. For example assuming stop masses to be around 1 TeV and maximal mixing the stop sector, one attains an upper bound on the light Higgs mass as:

$$m_h \stackrel{<}{\approx} 135 \text{ GeV}. \qquad (69)$$

7. Feynman Rules

In this section, we have written down all the mass matrices of the superpartners, their eigenvalues and finally the eigenvectors which are required to transform the superpartners in to their physical basis. The feynman rules corresponding to the various vertices have to be written down in this basis. Thus various soft supersymmetry breaking and supersymmetry conserving parameters entering these mass matrices would now determine these couplings as well as the masses, which in turn determine the strength of various physical processes like crosssections and decay rates. A complete list of the Feynman rules in the mass basis can be found in various references like Physics Reports like Haber & Kane [29] and D Chung et. al[34] and also in textbooks like Sparticles [30] and Baer & Tata [35]. A complete set of Feynman rules is out of reach of this set of lectures. Here I will just present two examples to illustrate the points I have been making here.

Due to the mixing between the fermionic partners of the gauge bosons and the fermionic partners of the Higgs bosons, the gauge and the yukawa vertices get mixed in MSSM. We will present here
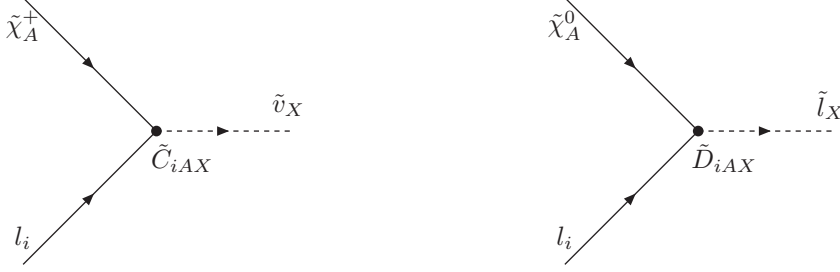
26

FIG. 3: lepton-slepton-chargino and lepton-slepton-neutralino vertices.

the vertices of fermion-sfermion-chargino and fermion-sfermion-neutralino where this is evident. These are presented in Figure 3.

(i) Fermion-Sfermion-Chargino :

This is the first vertex on the left of the figure. The explicit structure of this vertex is given by:

$$\tilde{C}_{iAX} = C^R_{iAX} P_R + C^L_{iAX} P_L \tag{70}$$

where $P_L(P_R)$ are the project operators[21] and C^R and C^L are given by

$$c^R_{iAX} = -g_2 (U)_{A1} R^\nu_{Xi} \tag{71}$$

$$C^L_{iAX} = g_2 \frac{m_{l_i}}{\sqrt{2} m_W \cos\beta} (V)_{A2} R^\nu_{Xi} \tag{72}$$

In the above U and V are the diagonalising matrices of chargino mass matrix M_C, R^ν is the diagonalising matrix of the sneutrino mass matrix, $M^2_{\tilde{\nu}}$. And the indices A and X runs over the dimensions of the respective matrices ($A = 1, 2$ for Charginos, $X = 1, 2, 3$ for sneutrinos), whereas i as usual runs over the generations, m_{l_i} is the mass of the i th lepton and rest of the parameters carry the standard definitions.

(ii) Fermion-Sfermion-Neutralino :

In a similar manner, the fermion-sfermion-neutralino vertex is given by:

$$\tilde{D}_{iAX} = D^R_{iAX} P_R + D^L_{iAX} P_L \tag{73}$$

where D^L and D^R have the following forms:

$$D^R_{iAX} = -\frac{g_2}{\sqrt{2}} \left\{ [-N_{A2} - N_{A1} \tan\theta_W] R^l_{Xi} + \frac{m_{l_i}}{m_W \cos\beta} N_{A3} R^l_{X,i+3} \right\} \tag{74}$$

$$D^L_{iAX} = -\frac{g_2}{\sqrt{2}} \left\{ \frac{m_{l_i}}{m_W \cos\beta} N_{A3} R^l_{Xi} + 2 N_{A1} \tan\theta_W R^l_{X,i+3} \right\} \tag{75}$$

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[21] $P_L = (1 - \gamma_5)/2$ and $P_R = (1 + \gamma_5)/2$.

In the above N is diagonalising matrices of neutralino mass matrix M_N, R^l is the diagonalising matrix of the slepton mass matrix, $M_{\tilde{l}}^2$. And the indices A and X runs over the dimensions of the respective matrices ($A = 1, .., 4$ for neutralinos, $X = 1, .., 6$ for sleptons), whereas i as usual runs over the generations.

8. *Think it Over:*

The LEP experiment at CERN searched for a light Higgs boson which has SM like couplings through the process $e^+ e^- \rightarrow ZH$ and has a put a limit on the lightest Higgs boson mass as $m_h \gtrsim 114.2\text{GeV}$. This limit applies to the light Higgs boson of the MSSM (except in some range and in the presence of CP violation in the Higgs sector). Take the formula of the 1-loop Higgs mass given by eq.(68) and simplify it by assuming the stop masses are of the similar order M_S and the mixing between the stops is maximal. Find out what is the least value of the M_S which is consistent with the Higgs mass. Now compute the 1-loop corrections to the minimisation conditions and check what is the amount of fine-tuning required to obtain the correct M_Z mass. Show that a few percent fine tuning is already required to satisfy the LEP limit on the light Higgs mass. The fine tuning rapidly increases with increasing Higgs mass. This goes under the name *Little Hierarchy Problem*.

VIII. 'STANDARD' MODELS OF SUPERSYMMETRY BREAKING

So far we have included supersymmetry breaking within the MSSM through a set of explicit supersymmetry breaking soft terms however, at a more fundamental we would like to understand the origins of these soft terms as coming from a theory where supersymmetry is spontaneously broken. In a previous section, we have mentioned that supersymmetry needs to be broken spontaneously in a hidden sector and then communicated to the visible sector through a messenger sector. In the below we will consider two main models for the messenger sector (a) the gravitational interactions and (b) the gauge interactions. But before we proceed to list problems with the general form soft supersymmetry breaking terms as discussed in the previous section. This is essential to understand what kind of constructions of supersymmetric breaking models are likely to be realised in Nature and thus are consistent with phenomenology.

The way we have parameterised supersymmetry breaking in the MSSM, using a set of gauge invariant soft terms, at the first sight, seems to be the most natural thing to do in the absence

of a complete theory of supersymmetry breaking. However, this approach is itselves laden with problems as we realise once we start confronting this model with phenomenology. The two main problems can be listed as below:

(i). Large number of parameters

Compared to the SM, in MSSM, we have a set of more than 50 new particles; writing down all possible gauge invariant and supersymmetry breaking soft terms, limits the number of possible terms to about 105. All these terms are completely arbitrary, there is no theoretical input on their magnitudes, relative strengths, in short there is no theoretical guiding principle about these terms. Given that these are large in number, they can significantly effect the phenomenology. In fact, the MSSM in its softly broken form seems to have lost predictive power except to say that there are some new particles within a broad range in mass(energy) scale. The main culprit being the large dimensional parameter space ~ 105 dimensional space which determines the couplings of the supersymmetric particles and their the masses. If there is a model of supersymmetry breaking which can act as a guiding principle and reduce the number of free parameters of the MSSM, it would only make MSSM more predictive.

(ii). Large Flavour and CP violations. As mentioned previously, the soft mass terms m_{ij}^2 and the trilinear scalar couplings A_{ijk} can violate flavour. This gives us new flavour violating structures beyond the standard CKM structure of the quark sector which can also be incorporated in the MSSM. Furthermore, all these couplings can also be complex and thus could serve as new sources of CP violation in addition to the CKM phase present in the Standard Model. Given that all these terms arbitrary and could be of any magnitude close to weak scale, these terms can contribute dominantly compared to the SM amplitudes to various flavour violating processes at the weak scale, like flavour violating decays like $b \to s + \gamma$ or flavour oscillations like $K^0 \leftrightarrow \bar{K}^0$ etc and even flavour violating decays which do not have any Standard Model counterparts like $\mu \to e + \gamma$ etc. The CP violating phases can also contribute to electric dipole moments (EDM)s which are precisely measured at experiments.

To analyse the phenomenological impact of these processes on these terms, an useful and powerful tool is the so called Mass Insertion (MI) approximation. In this approximation, we use flavour diagonal gaugino vertices and the flavour changing is encoded in non-diagonal sfermion propagators. These propagators are then expanded assuming that the flavour changing parts are much smaller than the flavour diagonal ones. In this way we can isolate the relevant elements of the sfermion mass matrix for a given flavour changing process and it is not necessary to analyse the full 6×6 sfermion mass matrix. Using this method, the experimental limits lead to upper bounds

on the parameters (or combinations of) $\delta_{ij}^f \equiv \Delta_{ij}^f/m_{\tilde{f}}^2$, known as mass insertions; where Δ_{ij}^f is the flavour-violating off-diagonal entry appearing in the $f = (u, d, l)$ sfermion mass matrices and $m_{\tilde{f}}^2$ is the average sfermion mass. In addition, the mass-insertions are further sub-divided into LL/LR/RL/RR types, labeled by the chirality of the corresponding SM fermions. The limits on various δ's coming from various flavour violating processes have been computed and tabulate in the literature and can be found for instance in Ref.[36].

These limits show that the flavour violating terms should be typically at least a couple of orders of magnitude suppressed compared to the flavour conserving soft terms[22]. While this is true for the first two generations of soft terms, the recent results from B-factories have started constraining flavour violating terms involving the third generation too. In light of this stringent constraint, it is more plausible to think that the fundamental supersymmetry breaking mechanism some how suppresses these flavour violating entries. Similarly, this mechanism should also reduce the number of parameters such that the MSSM could be easily be confronted with phenomenology and make it more predictive. We will consider two such models of supersymmetry breaking below which will use two different kinds of messenger sectors.

A. Minimal Supergravity

In the minimal supergravity framework, gravitational interactions play the role of messenger sector. Supersymmetry is broken spontaneously in the hidden sector. This information is communicated to the MSSM sector through gravitational sector leading to the soft terms. Since gravitational interactions play an important role only at very high energies, $M_p \sim O(10^{19})$ GeV, the breaking information is passed on to the visible sector only at those scales. The strength of the soft terms is characterised roughly by, $m_{\tilde{f}}^2 \approx M_S^2/M_{planck}$, where M_S is the scale of supersymmetry breaking. These masses can be comparable to weak scale for $M_S \sim 10^{10}$ GeV. This M_S^2 can correspond to the F-term vev of the Hidden sector. The above mechanism of supersymmetry breaking is called supergravity (SUGRA) mediated supersymmetry breaking.

A particular class of supergravity mediated supersymmetry breaking models are those which go under the name of "minimal" supergravity. This model has special features that it reduces to total number of free parameters determining the entire soft spectrum to five. Furthermore, it also removes the dangerous flavour violating soft terms in the MSSM. The classic features of this model

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[22] The flavour problem could also be alleviated by considering decoupling soft masses or alignment mechanisms.

are the following boundary conditions to the soft terms at the high scale $\sim M_{Planck}$:

- All the gaugino mass terms are equal at the high scale.

$$M_1 = M_2 = M_3 = M_{1/2}$$

- All the scalar mass terms at the high scale are equal.

$$m^2_{\phi_{ij}} = m^2_0 \delta_{ij}$$

- All the trilinear scalar interactions are equal at the high scale.

$$A_{ijk} = A h_{ijk}$$

- All bilinear scalar interactions are equal at the high scale.

$$B_{ij} = B$$

Using these boundary conditions, one evolves the soft terms to the weak scale using renormalisation group equations. It is possible to construct supergravity models which can give rise to such kind of strong universality in soft terms close to Planck scale. This would require the Kahler potential of the theory to be of the canonical form. As mentioned earlier, the advantage of this model is that it drastically reduces the number of parameters of the theory to about five, m_0, M (or equivalently M_2), ratio of the *vevs* of the two Higgs, $\tan\beta$, A, B. Thus, these models are also known as 'Constrained' MSSM in literature. The supersymmetric mass spectrum of these models has been extensively studied in literature. The Lightest Supersymmetric Particle (LSP) is mostly a neutralino in this case.

B. Gauge Mediated Supersymmetry breaking

In a more generic case, the Kahler potential need not have the required canonical form. In particular, most low energy effective supergravities from string theories do not posses such a Kahler potential. In such a case, large FCNC's and again large number of parameters are expected from supergravity theories. An alternative mechanism has been proposed which tries to avoid these problems in a natural way. The key idea is to use gauge interactions instead of gravity to mediate the supersymmetry breaking from the hidden (also called secluded sector sometimes) to the visible

MSSM sector. In this case supersymmetry breaking can be communicated at much lower energies ~ 100 TeV.

A typical model would contain a susy breaking sector called 'messenger sector' which contains a set of superfields transforming under a gauge group which 'contains' G_{SM}. Supersymmetry is broken spontaneously in this sector and this breaking information is passed on to the ordinary sector through gauge bosons and their fermionic partners in loops. The end-effect of this mechanism also is to add the soft terms in to the lagrangian. But now these soft terms are flavour diagonal as they are generated by gauge interactions. The soft terms at the messenger scale also have simple expressions in terms of the susy breaking parameters. In addition, in minimal models of gauge mediated supersymmetry breaking, only one parameter can essentially determine the entire soft spectrum.

In a similar manner as in the above, the low energy susy spectrum is determined by the RG scaling of the soft parameters. But now the high scale is around 100 TeV instead of M_{GUT} as in the previous case. The mass spectrum of these models has been studied in many papers. The lightest supersymmetric particle in this case is mostly the gravitino in contrast to the mSUGRA case.

1. *Think it Over*

- In both gravity mediated as well as gauge mediated supersymmetry breaking models, we have seen that RG running effects have to included to study the soft terms at the weak scale. Typically, the soft masses which appear at those scales are positive at the high scale. But radiative corrections can significantly modify the low scale values of these parameters; in particular, making one of the Higgs mass to be negative at the weak scale leading to spontaneous breaking of electroweak symmetry. This mechanism is called radiative electroweak symmetry breaking. Consider two hypothetical situations when (a) the top mass is twice its present value $m_t = 2\ m_t$ (b) the top mass is 1/10 th its present value $m_t = m_t/10$. In which case there would be more efficient Electroweak symmetry breaking ?

- The recent limits from LHC already put severe constraints on the lightest squarks and gluino masses. They push their masses to be greater than 800 GeV - 1 TeV. In fact, this has severe constraints on mSUGRA model. For latest limits have a look at [15].

IX. REMARKS

The present set of lectures are only a set of elementary introduction to the MSSM. More detailed accounts can be found in various references which we have listed at various places in the text. In preparing for these set of lectures, I have greatly benefitted from various review articles and text books. I have already listed some of them at various places in the text. Martin's review [37] is perhaps the most comprehensive and popular references. It is also constantly updated. Some other excellent reviews are [38] and [39]. A concise introduction can also be found in [40]. For more formal aspects of supersymmetry including a good introduction to supergravity please have a look at [41] and [42]. For Grand Unified theories and supersymmetry, please have a look at [43], [44], [45] and [46]. For a comprehensive introduction to supersymmetric dark matter, please see [47]. Finally, I would also recommend the original papers of anomaly mediated supersymmetry breaking [48]. Happy Susying.

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https://twiki.cern.ch/twiki/bin/view/Atlas/WebHome
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